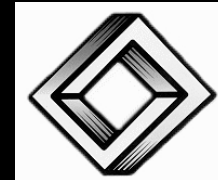
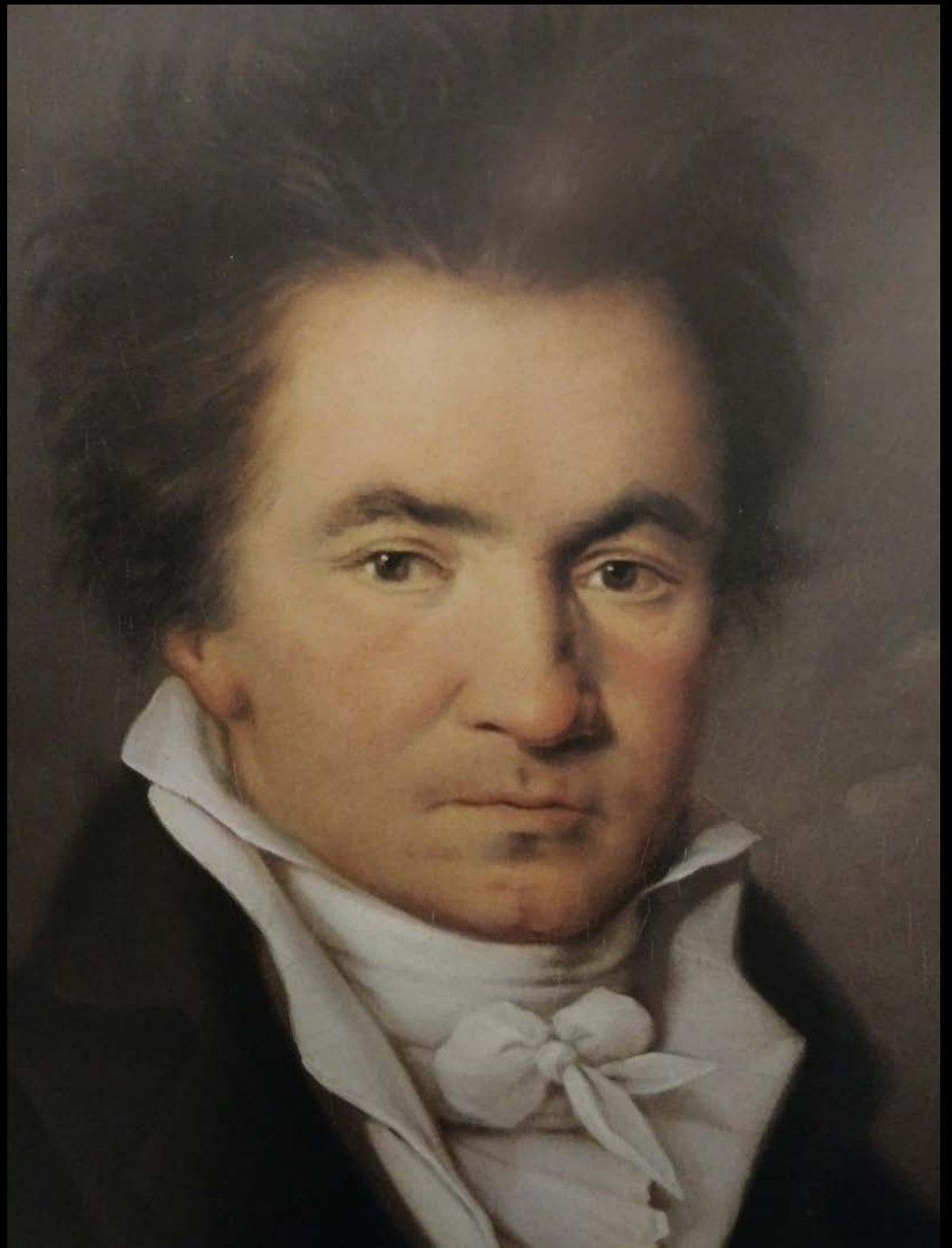


Beethoven: matemático intuitivo, a los 250 años de su natalicio.

Dr. Emilio Lluís-Puebla





Beethoven por W. J. Mahler (1815)
The Karajan Collection, Salsburg

BEETHOVEN

Klaviersonaten

BAND I

URTEXT



BEETHOVEN

Klaviersonaten

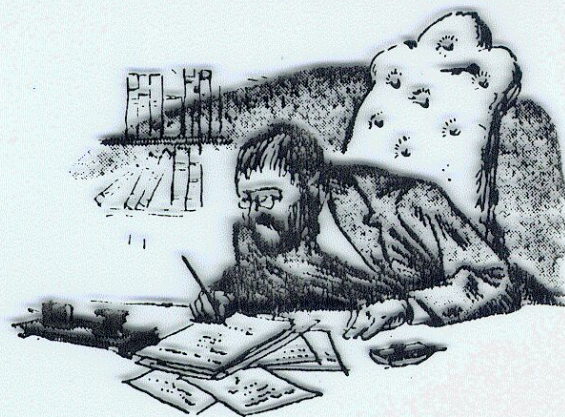
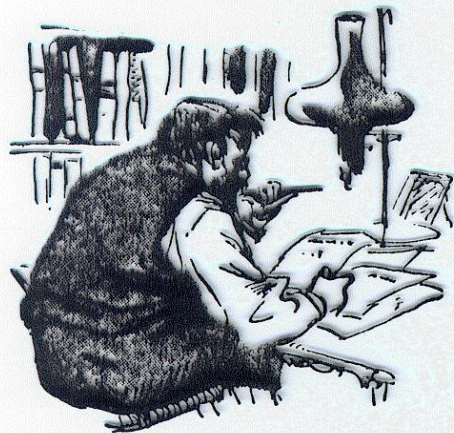
BAND II

URTEXT





Henri Poincaré
(1854-1912)



RAE

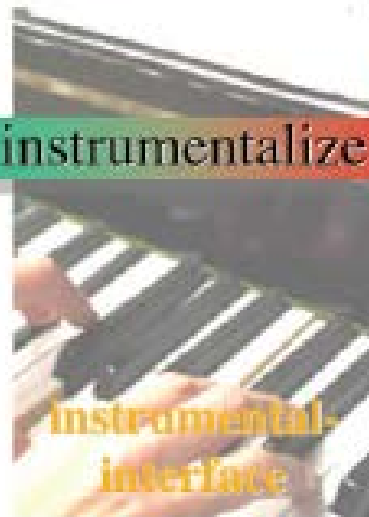
gesto. (Del lat. gestus).

- 1. m. Movimiento del rostro, de las manos o de otras partes del cuerpo con que se expresan diversos afectos del ánimo.**
- 2. m. Movimiento exagerado del rostro por hábito o enfermedad.**
- 3. m. Contorsión burlesca del rostro.**
- 4. m. Semblante, cara, rostro.**
- 5. m. Acto o hecho.**
- 6. m. Rasgo notable de carácter o de conducta.**
- 7. m. ant. Aspecto o apariencia que tienen algunas cosas inanimadas.**

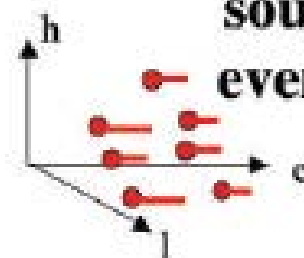
gestures



instrumentalize



sound events



may have happened...
particular...
was not...
any time...



thaw

freeze

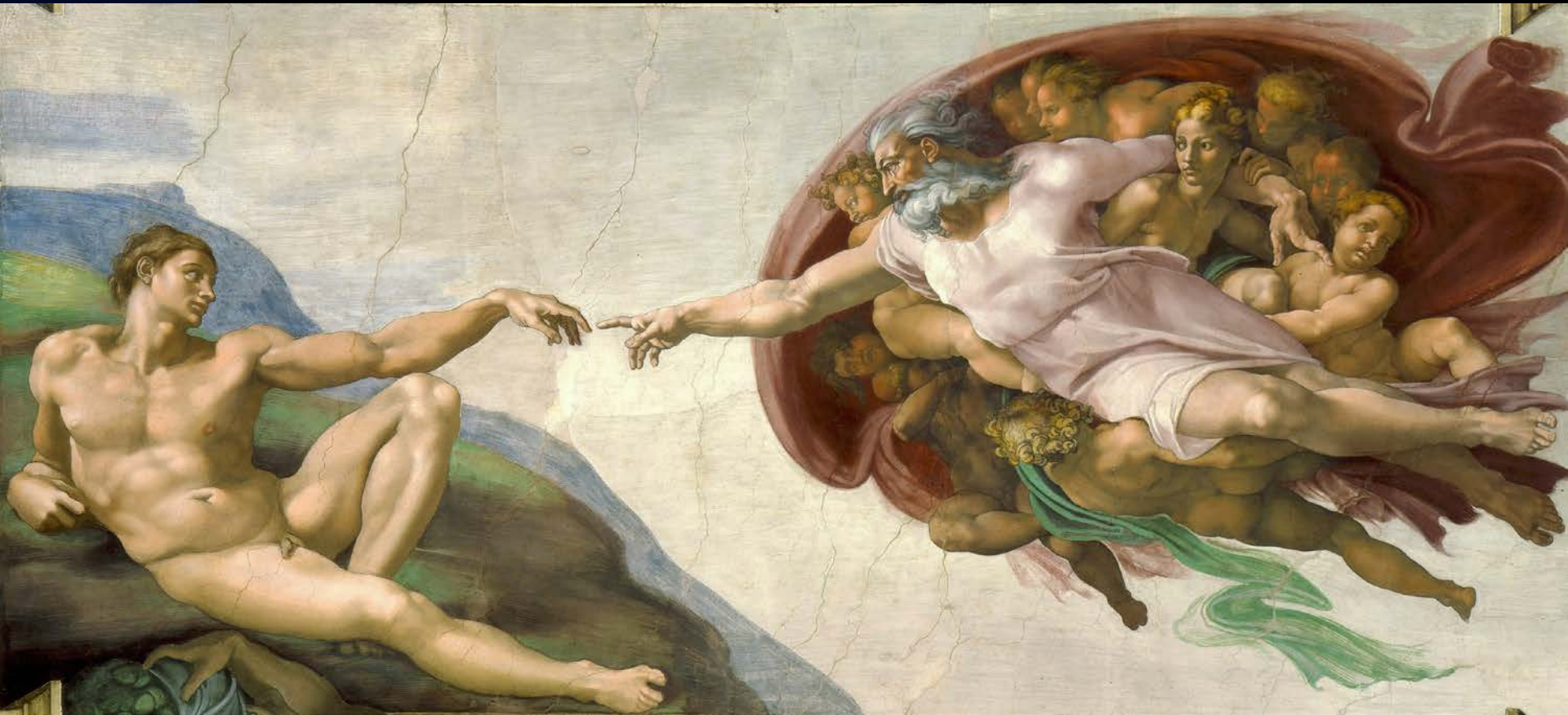
score



ϕ



analysis



Matemáticamente, se define un gesto como un grafo o gráfica dirigida D , llamado el *esqueleto* del gesto

Junto con una transformación g que asocia a cada flecha a de D una curva continua $g(a): I \rightarrow X$

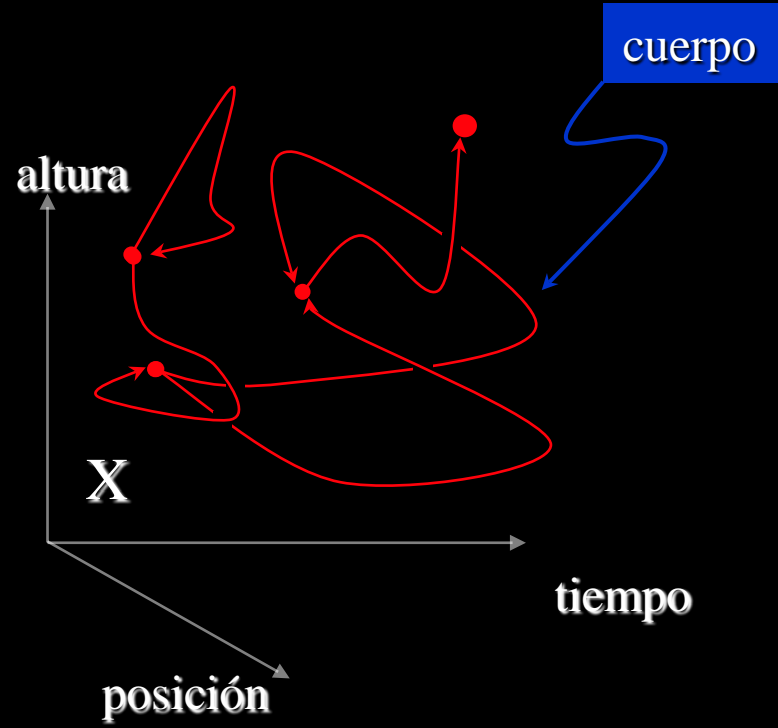
Cuerpo del gesto

Gesto = morfismo $g: \mathbf{D} \rightarrow \vec{\mathbf{X}}$
 de digráficas con valores en una
gráfica espacial dirigida $\vec{\mathbf{X}}$ de un espacio topológico \mathbf{X}
 (= gráfica dirigida de curvas continuas en \mathbf{X})

esqueleto



g



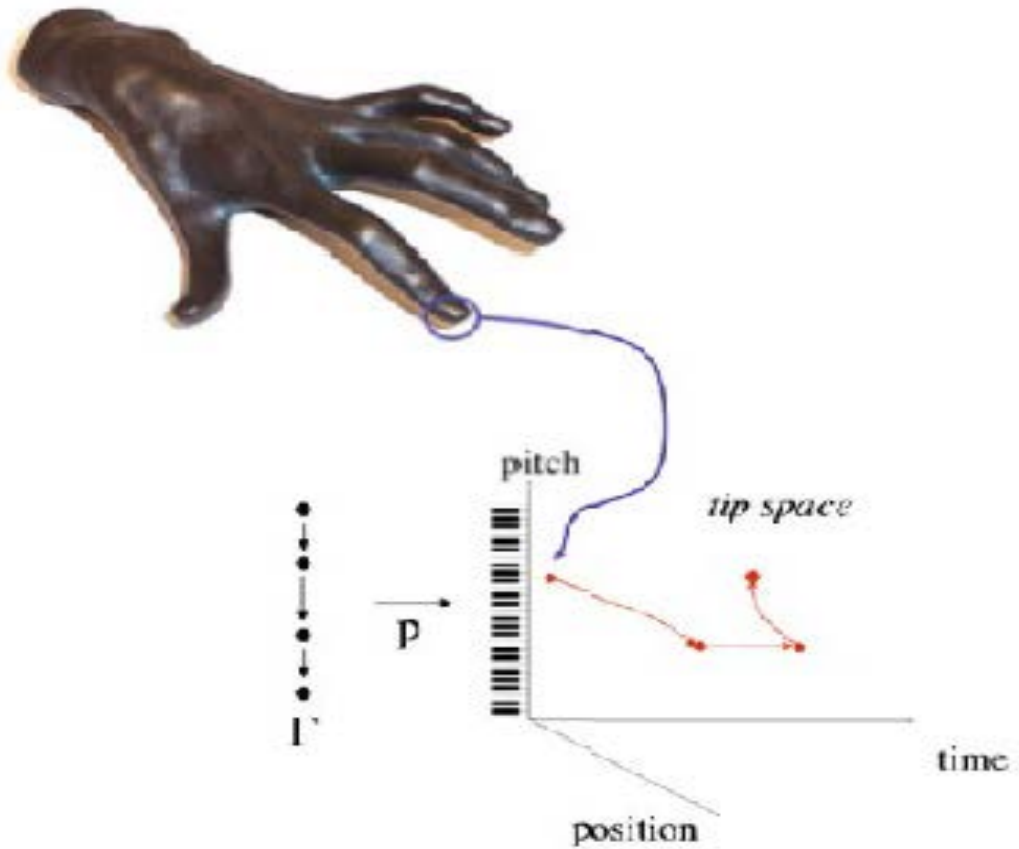


Fig. 33.12. An elementary fingertip gesture.

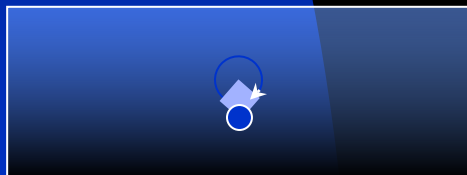
$$\begin{array}{ccc}
 [1] & \xrightarrow{q \circ a^*} & \vec{X} \\
 a^* \downarrow & & \downarrow \overrightarrow{I d_X} \\
 \Delta & \xrightarrow{g} & \vec{X}
 \end{array}$$

$\text{Digraf}(\Delta, X) \cong$ espacio topológico de gestos
con esqueleto Δ y cuerpo en X
notación: $\Delta @ X \rightarrow$

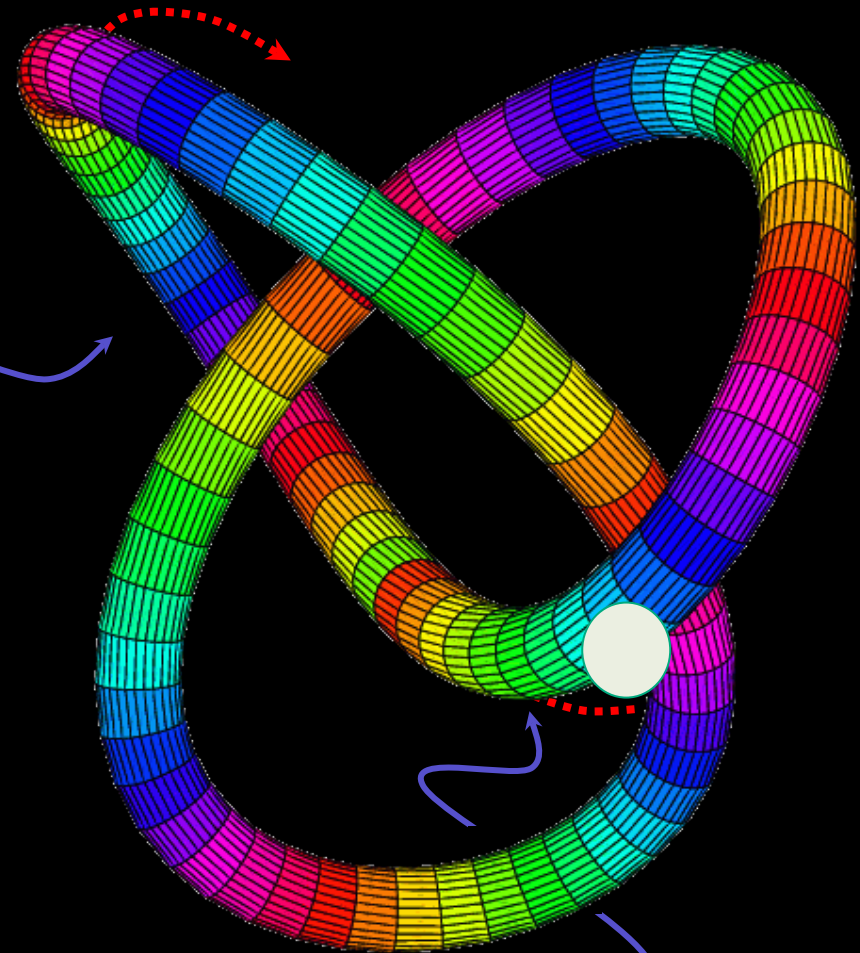
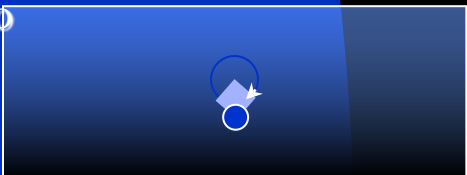
Hipergestos

„Lazo de lazos“

nudo



círculo





Teorema (Escher) de Guerino Mazzola:

Dado un espacio topológico X , una sucesión de digráficas

$$\Phi_1, \Phi_2, \dots, \Phi_n$$

y una permutación π de $1, 2, \dots, n$,

existe un homeomorfismo

$$\Phi_1 @ \dots @ \Phi_n @ X \approx \Phi_{\pi(1)} @ \dots @ \Phi_{\pi(n)} @ X$$

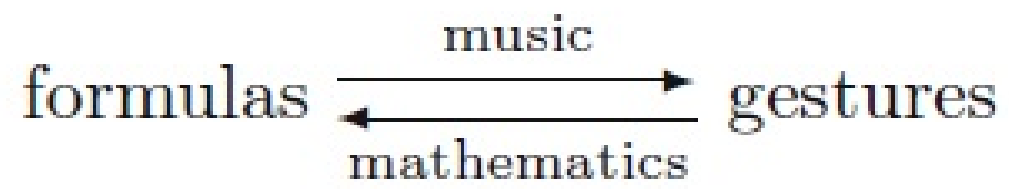
Demostración: ver tesis de licenciatura en matemática de mi alumna Yemile Chávez (2015)

Usando el Teorema de Escher, tenemos homomorfismos frontera

$$\partial_n: C_n(X, \Phi_*) \rightarrow C_{n-1}(X, \Phi_*)$$

para cualquier sucesión Φ_* de digráficas, generalizando $\uparrow \uparrow \dots \uparrow$, y $\partial^2 = 0$, así que tenemos módulos de homología

$$H_n = \text{Ker}(\partial_n) / \text{Im}(\partial_{n+1}).$$



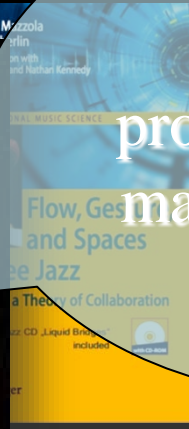
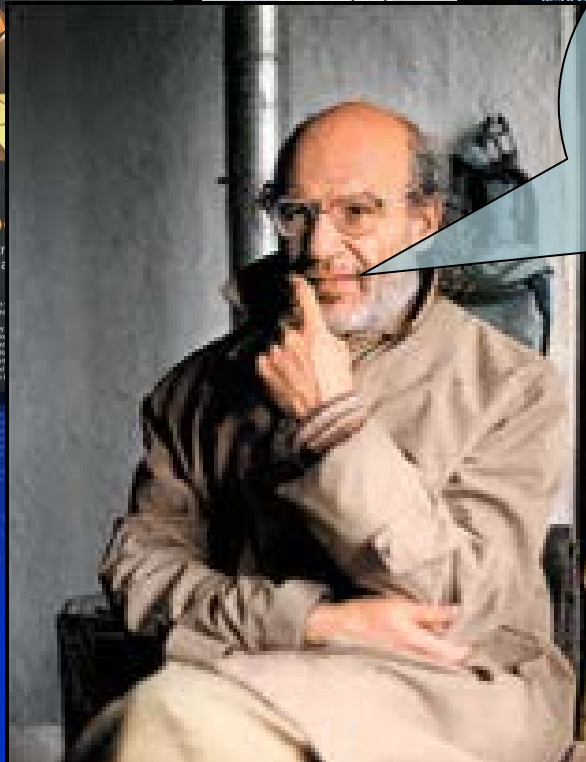
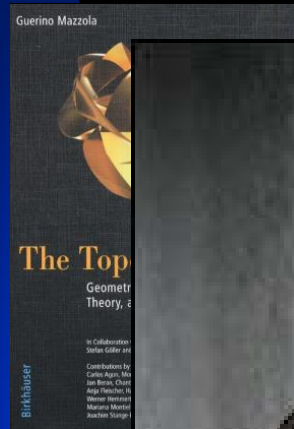
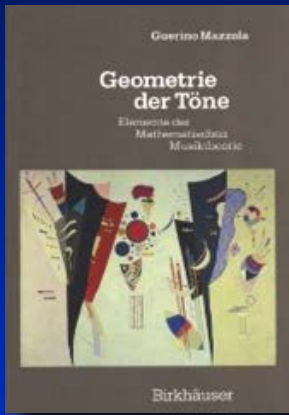
Música

Fórmulas \longleftrightarrow **Gestos**

Matemática

Alexander
Grothendieck:

„Esta es
probablemente la
matemática de la
nueva era“



Computational Music Science



Guerino Mazzola
Maria Mannone
Yan Pang

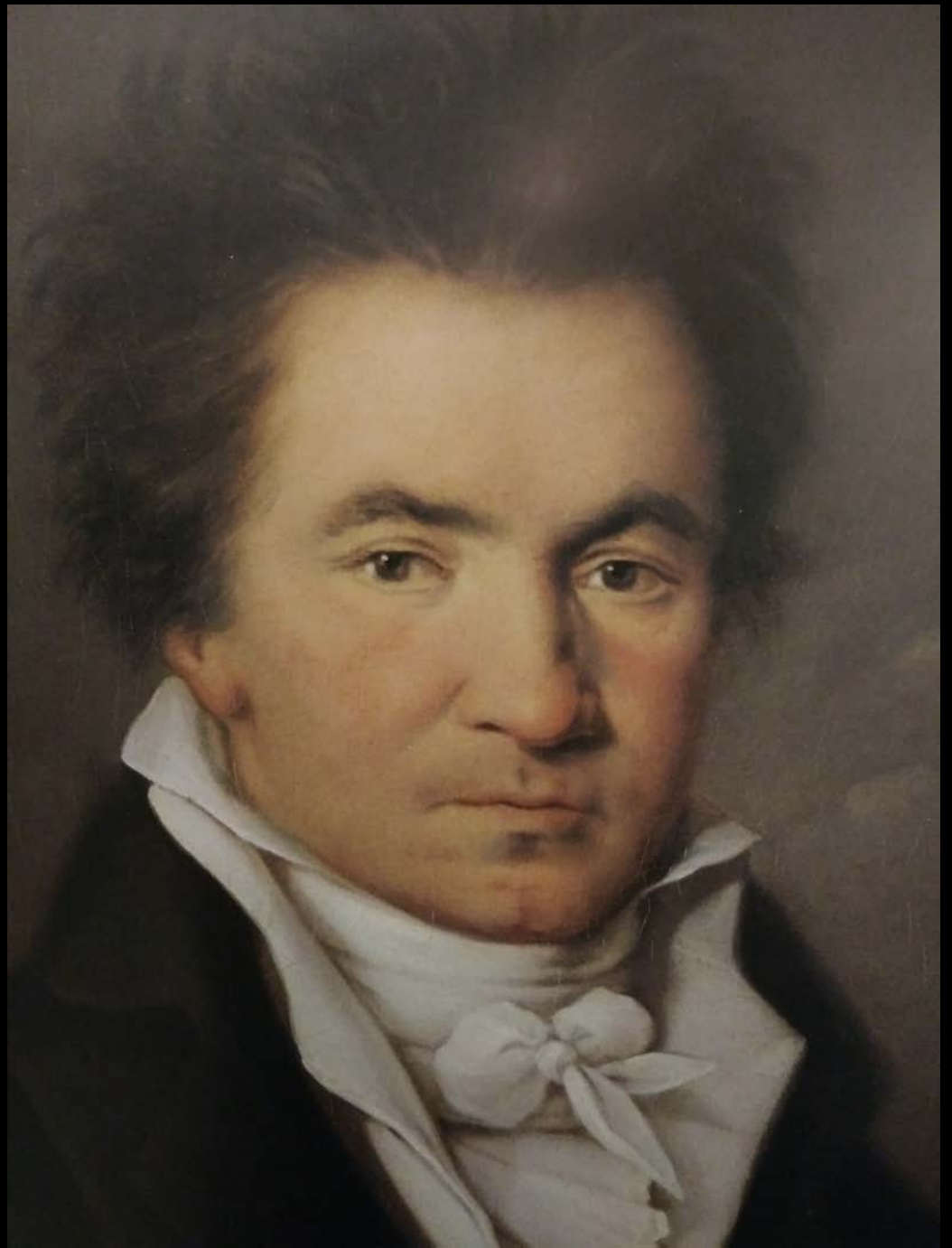


Cool Math for Hot Music

A First Introduction to Mathematics
for Music Theorists

 Springer

¿Para quién componía Beethoven?



Beethoven por W. J. Mahler (1815)
The Karajan Collection, Salsburg

Inversion_{d_b} : G⁽³⁾ → E_b⁽³⁾

124 *pp* *sempre pp*

126 *cresc.* *f* *f* *p*

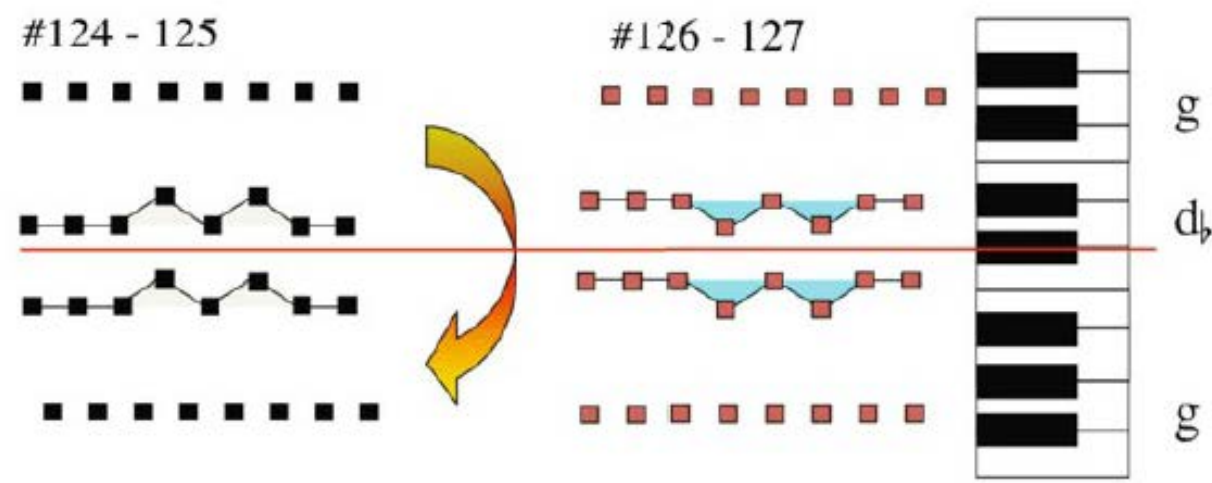
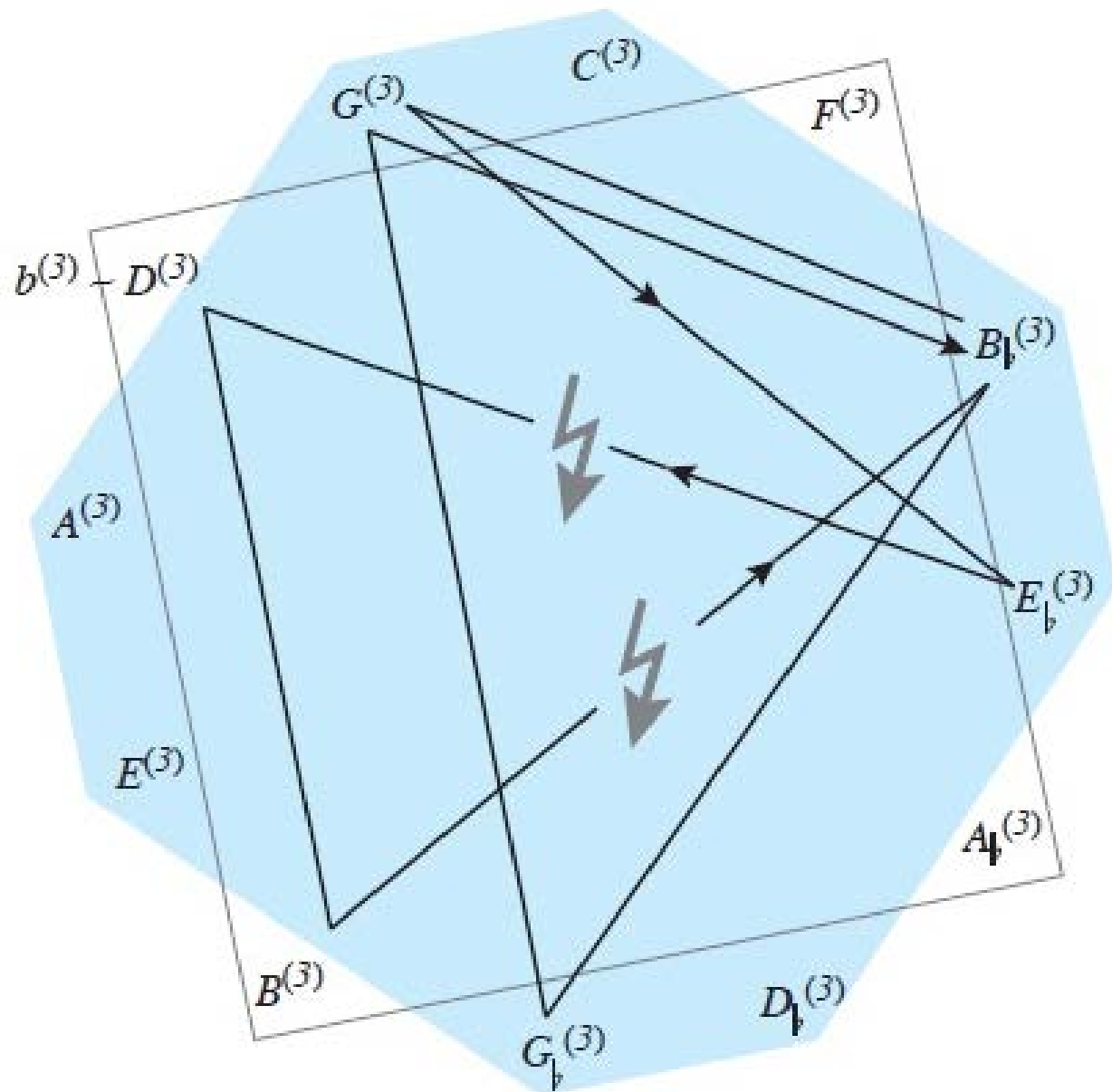


Fig. 23.6. A complex modulation $G \rightarrow E_b$.



Guerino Mazzola



The Topos of Music

Geometric Logic of Concepts,
Theory, and Performance

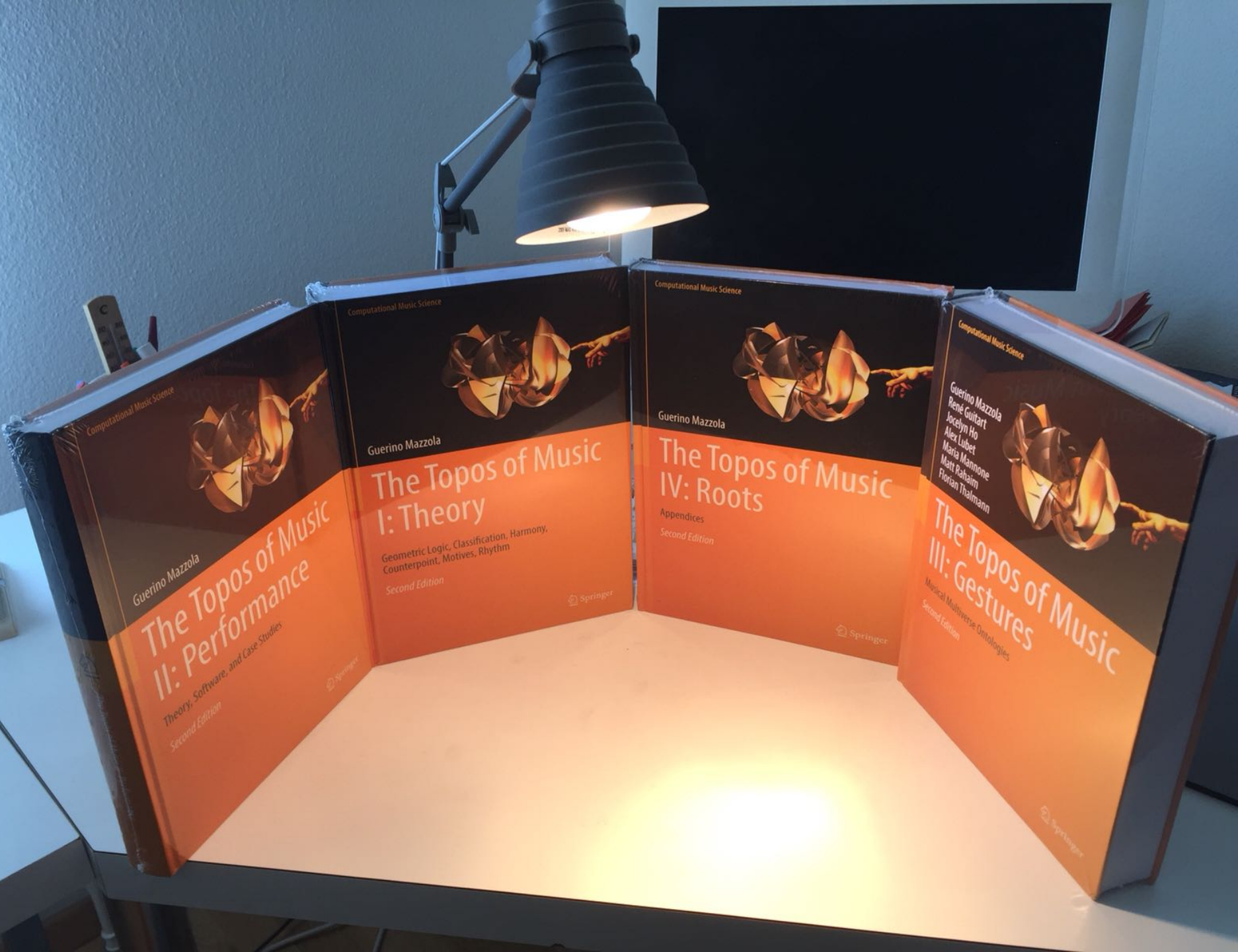
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Stefan Göller and Stefan Müller

Contributions by
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Jan Beran, Chantal Buteau, Roberto Ferretti,
Anja Fleischer, Harald Friepertinger, Jörg Garbers,
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- Guerino Mazzola y Emilio Luis-Puebla en 1997



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The Future of Music

Towards a Computational Musical
Theory of Everything

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SONATE

Maximiliane Brentano geoidmet
Komponiert 1820

273

Opus 109

Vivace, ma non troppo.

sempre legato

30.

Adagio espressivo

V_1, V_2, \dots, V_6 están todas dirigidas hacia el tema X , un hecho que escribiríamos como si esta configuración estuviera incrustada en una categoría, es decir, por seis morfismos (perspectivas sobre X , tomadas de las direcciones V_1, V_2, \dots, V_6)

$$f_1: V_1 \rightarrow X, \quad f_2: V_2 \rightarrow X, \quad \dots, \quad f_6: V_6 \rightarrow X.$$

1.- Una variación melódica

f1: V1 → X

construida sobre un entorno homofónico con acompañamiento de vals.

In a song-like way,
Gesangvoll, mit innigster Empfindung
Andante molto cantabile ed espressivo 56 Arr

mezza voce

cresc. p

cresc. sf mezza voce

Var. I
Molto espressivo 60 Arr

cresc.

1. 2. 3. 2. cresc.

1. 2. cresc.

281

Una variación rítmica

$f_2: V_2 \rightarrow X$

con cambios sofisticados de los tonos del tema en una secuencia de arpeggios interválicos.

282 Var. II 69 Arr 63 Sch
Leggiermente

The image shows a page of musical notation for Variation II, numbered 282. The title is "Var. II 69 Arr 63 Sch" and the tempo marking is "Leggiermente". The score is written for piano and consists of five systems of music. The first system (measures 31-34) begins with a piano (*p*) dynamic. The second system (measures 35-38) includes a crescendo (*cresc.*) marking. The third system (measures 39-42) features dynamic markings of *dimin.*, *cresc.*, *dimin.*, and *p*. The fourth system (measures 43-46) is marked *teneramente* and includes trills (*tr*) and a *rit. - un poco* instruction. The fifth system (measures 47-50) also includes trills (*tr*). The key signature is three sharps (F#, C#, G#) and the time signature is 3/4.

Una variación contrapuntística

$f_3: V_3 \rightarrow X$

construida sobre correspondencias por inversiones del tema y su redistribución en voces diferentes.

284 Var. III Allegro vivace

The image shows a page of a musical score for Variation III, Allegro vivace. The score is written for piano and bass clef staves. It consists of six systems of music. The first system is marked with a forte (f) dynamic. The second system is marked with piano (p) and crescendo (cresc.). The third system is marked with forte (f) and piano (p) dynamics. The fourth system is marked with piano (p) and crescendo (cresc.). The fifth system is marked with piano (p). The sixth system is marked with piano (p) and crescendo (cresc.). The score includes various musical notations such as notes, rests, and ornaments. There are also some handwritten annotations and markings on the score.

^{*)} In der Originalangabe in diesem Takt oben staccato (nicht in der Eigenschrift).

^{*)} Staccato above in this bar in the original edition (not in the autograph).

^{*)} En haut, dans cette mesure de l'édition originale: staccato (non dans l'autographe).

Una variación permutacional

$$f_4: V_4 \rightarrow X$$

intercambiando algunas notas del tema para captar todo el poder cantabile del tema en una interpretación parecida a una fughetta.

Var. IV
Etwas langsamer als das Thema *Adm*
Un poco meno andante cioè un poco più adagio come il tema

285

piacevole

cres. poco a poco

dim.

1.

2.

pp

Red

Esta variación

$f_5: V_5 \rightarrow X$

también funciona como permutacional, pero es dedicado a revelar el poder del tercero, que era también el intervalo inicial (g , e) del tema original.

286

108

109

110

111

112

113

114

115

116

117

118

Var. V

Allegro, ma non troppo

sempre pp

cresc.

f

il più forte

ff

dim.

dolce

pp

f

sempre f

*) Akzente u. *sf*-Zeichen in diesem Takt getreu nach Eigensch. ; in Orig. Ausg. (aus Schindler's Besitz mit Beethovens eigenh. Korrekturen) sind die beiden Akzente auf das folgende Sechzehntel verschoben.

**) Haltebogen von e^2-e^3 in der Eigensch. nicht dagegen in der Originalausgabe.

*) Accents & *sf* markings in this bar follow autogr. exactly; in orig. edition (from Schindler's Coll. with Beethoven's own corr.) the 2 accents have been shifted to next sixteenth-note (semiquaver).

**) Tie connecting e^2-e^3 in the autograph, but not in the original edition.

*) Dans cette mes. accents et signes de *sf* identiques à l'autogr.; dans l'éd. orig. (appartenant à Schindler et corrigée par Beethoven) les 2 accents ont été déplacés sur la double croche suiv.

**) Dans l'autographe, arc de liaison de e^2 à e^3 ; il manque, par contre, dans l'édition originale.

Var. VI
Tempo I del tema
Cantabile 56

158

159

160

162

164

colímite

colim (D) → X

161

Handwritten musical notation for measures 161 and 162. The system consists of a treble and bass staff. Measure 161 features a complex melodic line in the treble with many beamed notes and a steady eighth-note accompaniment in the bass. Measure 162 continues the melodic development with some chromaticism. A handwritten 'f' dynamic marking is present in measure 162.

163

Handwritten musical notation for measures 163 and 164. Measure 163 shows a continuation of the intricate melodic patterns in the treble. Measure 164 features a more active bass line with eighth-note accompaniment. A handwritten 'f' dynamic marking is present in measure 164.

165

Handwritten musical notation for measures 165 and 166. Measure 165 contains a dense texture of beamed notes in the treble. Measure 166 shows a similar texture with some chromatic movement in the bass line.

171

Handwritten musical notation for measures 171 and 172. Measure 171 begins with an '8' marking above the treble staff, indicating an octave shift. The treble part has a rhythmic pattern of eighth notes. Measure 172 continues this pattern with some chromaticism.

173

Handwritten musical notation for measures 173 and 174. Measure 173 starts with an '8' marking. Measure 174 features a treble staff with notes marked with '7' and '8' above them, and a bass staff with a complex accompaniment. A handwritten 'p' dynamic marking is present in measure 174.

175

Handwritten musical notation for measures 175 and 176. Measure 175 begins with an '8' marking and notes marked with '7' and '8'. Measure 176 shows a treble staff with notes marked with '7' and '8', and a bass staff with a complex accompaniment. A handwritten 'p' dynamic marking is present in measure 176.

291

129

130

131

132

133

134

135

136

137

138

139

140

dimin.

piu dimin.

pp

cantabile

cresc.

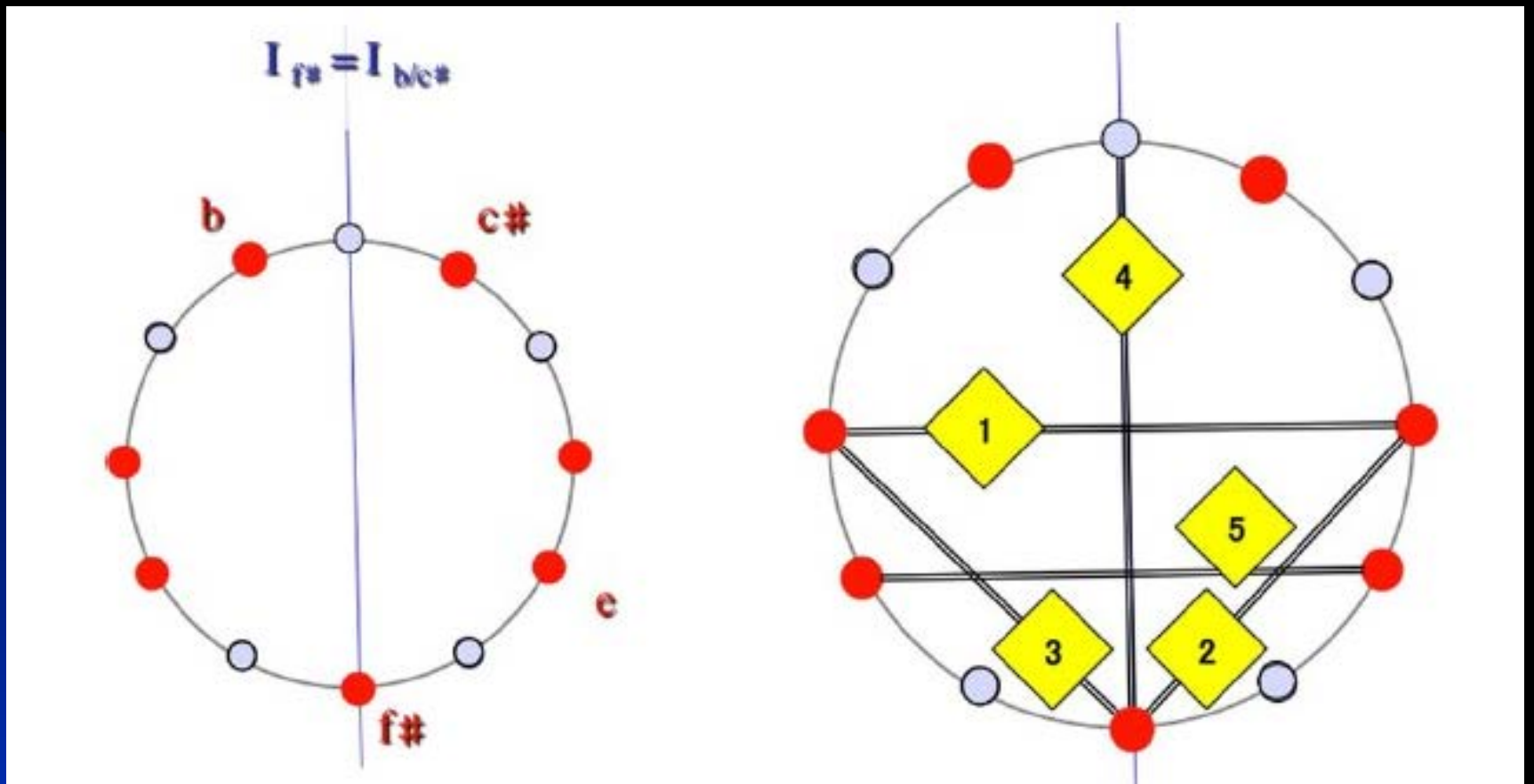
p

cresc.

sf

ritard.

*) Nach der Originalausgabe; in der Eigenschrift Akkad. *) According to orig. edition; in autogr. chord. *) D'après l'édition orig.; dans l'autogr. accord.



Izquierda: la simetría de inversión de mi mayor; Derecha: los intervalos arpegiados simétricamente ordenados 1, 2, 3, 4, 5 en los compases 173-174. Los intervalos 1, 4, 5 son simétricos y los intervalos 2, 3 se corresponden entre sí por la simetría de mi mayor.



Primer movimiento de la Sonata KV 545 en Do mayor de Mozart.
El trino d,e prepara la tónica en la función de cadencia.

m.173

m.174

The image shows a musical score for two measures, m.173 and m.174, in the key of D major. The notation is on a single treble clef staff. Measure 173 contains 12 notes, and measure 174 contains 12 notes. Below the staff, red and blue numbers (1-5) indicate fingerings for each note. Red numbers are used for notes in the first half of each measure, and blue numbers for notes in the second half. Arrows connect the numbers to the notes they apply to. Additionally, there are diagrams of arpeggios (triangles) drawn under the notes, with red triangles for the first half and blue triangles for the second half of each measure. A '8' is written above the first note of measure 174, possibly indicating an octave or a specific fingering technique.

La secuencia de arpeggios de intervalo está organizada con alta simetría con respecto a la simetría de Mi mayor $I_{f\#} = I_{b/c\#}$.

Computational Music Science

Guerino Mazzola
Joomi Park
Florian Thalmann



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Strategies and Tools in Composition
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A musical score for a single melodic line, showing a sequence of notes and rests on a staff.

A₁: melodic



A complex musical score representing the synthesis of multiple layers, showing multiple staves with various musical notations.

A₆: synthesis



A musical score showing a melodic line with specific intervals, likely thirds, illustrating a permutational structure.

A₅: permutational
with thirds



A musical score showing a rhythmic pattern, possibly a drum line or a rhythmic accompaniment, with notes and rests.

A₂: rhythmical



A central musical score, likely the result of the synthesis process, showing a complex arrangement of notes and rests.



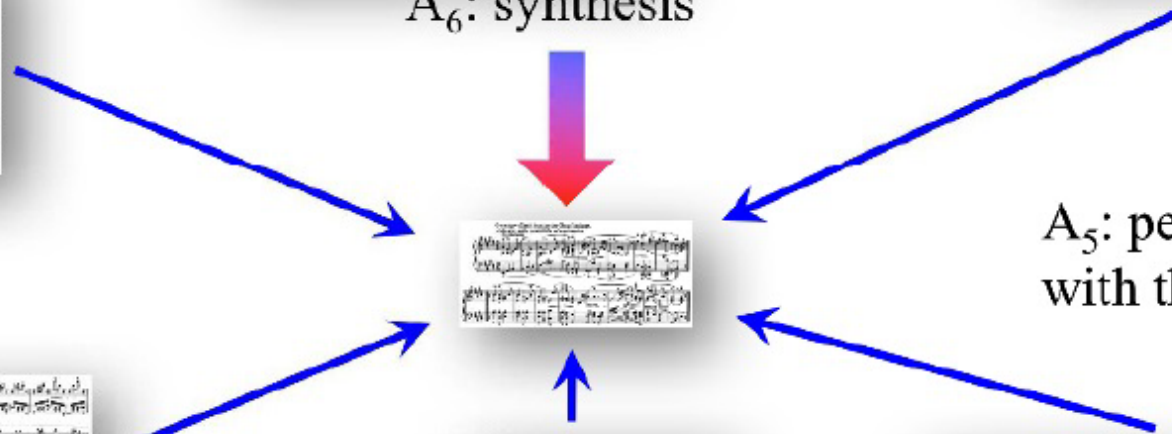
A musical score showing a contrapuntal structure, with multiple voices or parts moving independently.

A₃: contrapuntal



A musical score showing a permutational structure, with notes and rests arranged in a specific, repeating pattern.

A₄: permutational



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Teoría de la Expresión
Musical

Ejecución Inversa
Crítica Musical

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In Collaboration with
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Carl Czerny (1791-1857)

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Herausgeber / Edition
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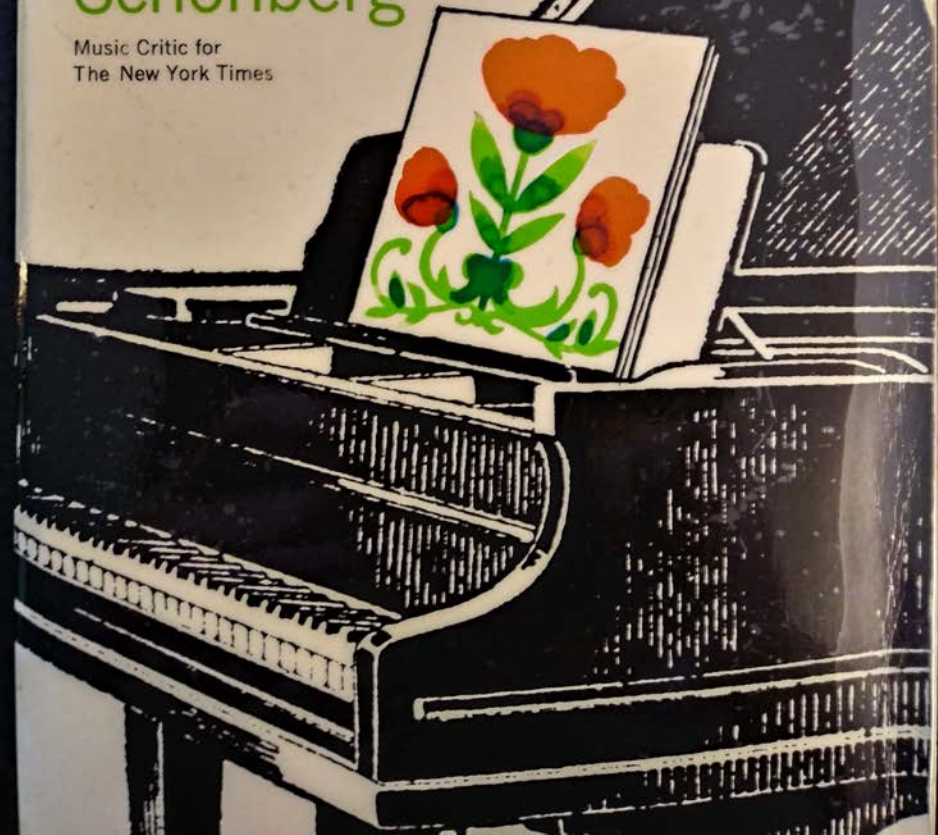
ON THE PROPER PERFORMANCE
OF ALL BEETHOVEN'S WORKS
FOR THE PIANO

CZERNY'S "Reminiscences of Beethoven"
and chapters II and III from volume IV of the
"Complete Theoretical and Practical Piano Forte School op.500"

Edited and with a Commentary by
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Music Critic for
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Música, Matemática y Concertismo

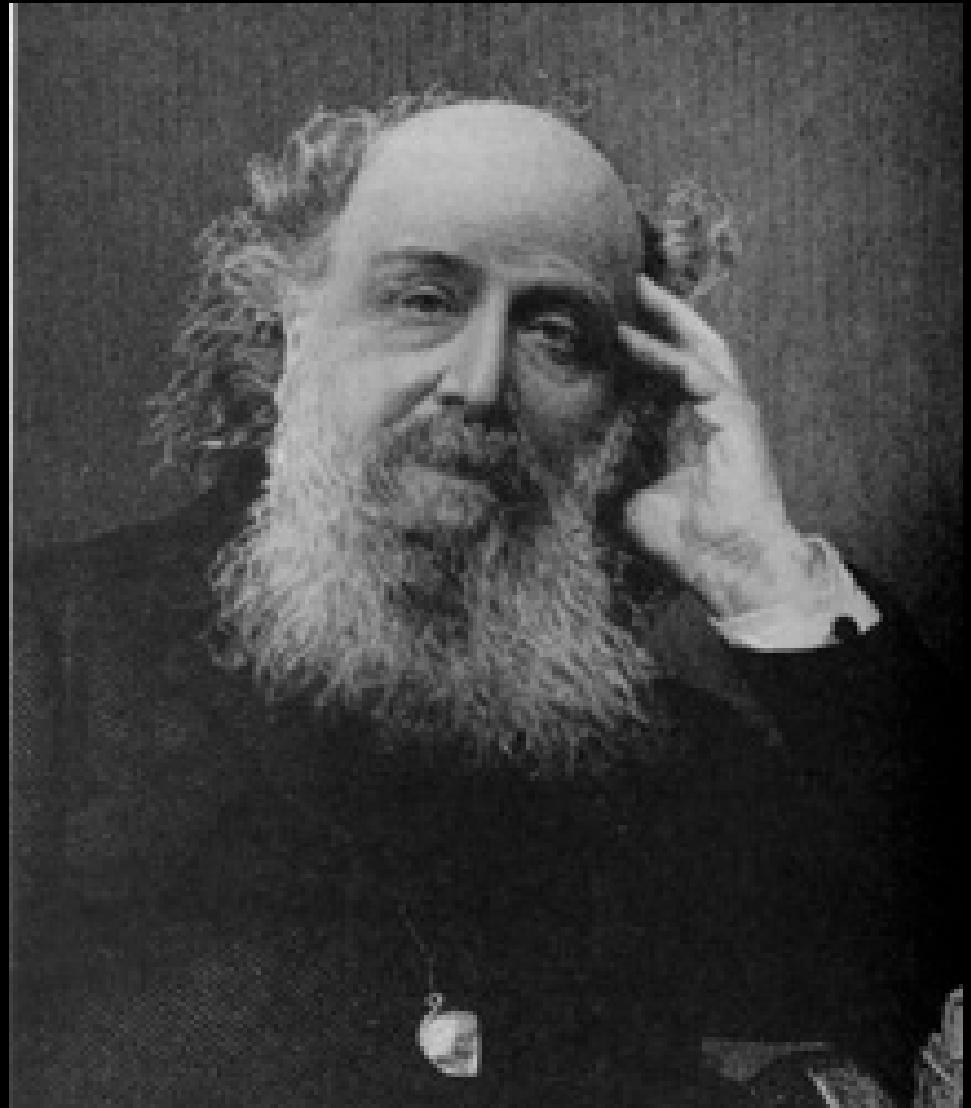
Emilio Lluís



Emilio Lluís-Puebla

**MÚSICA,
MATEMÁTICA
Y CONCERTISMO**

Publicaciones electrónicas del IMCH



Sir James Joseph Sylvester
(1814-1897)

*"May not Music be described as the
Mathematics of sense,
Mathematics as
Music of the reason?
The soul of each the same?"*

J. J. Sylvester

(1864)

¿Acaso no puede describirse la Música
como la Matemática de lo sensible
y la Matemática como la
Música del entendimiento?

El alma de cada una, la misma.

J. J. Sylvester

(1864)

La Matemática
es una de las Bellas Artes,
la más pura de ellas,
que tiene el don de ser
la más precisa
y la precisión de las Ciencias.

E. Lluís-Puebla

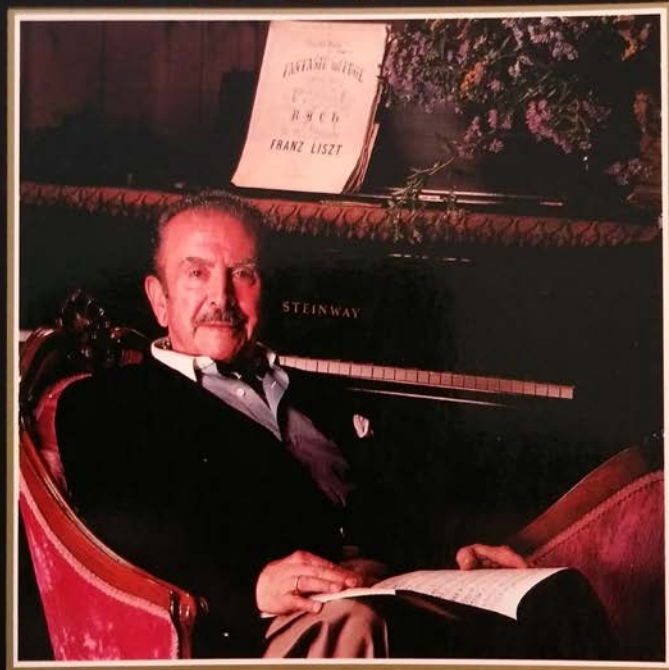


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